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Sound transmission in multilayered structures – Introducing finite structural connections in the transfer matrix method

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ABSTRACT

The transfer matrix technique is an efficient tool for calculating sound transmission through multilayered structures. Recent developments, as to the calculation of the sound reduction index, have given better fit to measurement data, e.g. due to the simulation of finite size samples by the spatial windowing technique. In practice, however, finite size structural connections (points or lines) normally do exist between leaves in wall constructions and these are not “natural” elements in the transfer matrix method. The paper describes a simple method to account for the effect of point- and line-connections in double-leaf constructions in a transfer matrix setup. To cover the frequency range above the critical frequency of the constituent plates, some new developments as to the forced radiation from plates were needed. Predicted results compare favorably with measurement results for a number of different cases, also including heavy walls.

1. INTRODUCTION

For predicting the diffuse field sound transmission through a single plane homogeneous structure, such as a partition between rooms, a number of formulas may be found in the literature, see e.g. reference[1]. Tools for predicting the sound transmission loss, specified by the sound reduction index, of general multilayered structures are normally found only for special cases. Double-leaf partitions in various forms have been and still are a current object for study due to their practical use, where the simplest type has been the one without structural connections. More often, information is needed as to the effect of structural connections such as line- or point-like bridges (studs or “ties”) between the leaves. An overview of the literature on this subject may be found e.g. in a paper by Wang et al.[2], a paper which presents a theoretical modeling of a lightweight double leaf partition stiffened by periodically placed studs. Periodic structure theory is applied as well as a “smeared” model, treating the studs as uniformly distributed elastic springs. The latter approach certainly exhibits deficiencies as compared with the periodic theory.

In this paper we will present a simpler approach, based on the transfer matrix technique with a modification based on the semi-empirical approach put forward by Sharp[3]. He treated the sound transmission through double panels with sound bridges by subtracting a correction term based on the sound power radiated by a point- or line-loaded panel. Certainly, no account is taken neither to the mass nor the finite stiffness of the bridges, they are considered to be infinitely stiff. This approach has been successfully

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applied e.g. by Lam[4] to double metal claddings with point connectors, however, not put into the framework of transfer matrices.

This paper will, in the section presenting the theory, first give a short outline of the model by Sharp[3], pointing out the need for information on the radiation factor (also called radiation efficiency) for point- and line-driven plates. A new development as to the radiated sound power from line-driven plates will be presented and thereby the radiation factor is implicitly given. Comparisons between predicted results, using this simplified model, and measurement results will be presented.

2. THEORY

A. Sound bridge correction

Following Sharp[3], the sound reduction index R of a double-leaf partition with sound bridges may be expressed as

$$R = R_P - \Delta R = R_P - 10\lg\left[1 + \frac{W_{2,B}}{W_{2,P}}\right], \tag{1}$$

where R_P is the sound reduction index of the double-leaf partition without the bridges and ΔR is the correction term due to the bridges. The power radiated from second plate is divided into two parts, $W_{2,P}$ and $W_{2,B}$, the power radiated without the bridges and the power radiated due to the action of the bridges, respectively. It may be shown that the power ratio in the correction term may be expressed as

$$\frac{W_{2,B}}{W_{2,P}} = n\sigma_B \cdot \left|\frac{v_B}{v_1}\right|^2 \cdot \left|\frac{v_1}{v_2}\right|^2 = n\sigma_B \cdot \left|\frac{Z_{B1}}{Z_{B1} + Z_{B2}}\right|^2 \cdot \left|\frac{v_1}{v_2}\right|^2, \tag{2}$$

where σ_B is the radiation factor of the second plate driven by one of the number n bridges acting over the partition area S. The second term contains the input impedances of the plates seen from the sound bridge. The last term is the squared ratio of the velocities of plate 1 and plate 2 in the absence of the bridges. Sharp[3] gives an approximate expression for this ratio but in the framework of transfer matrices there is no need to do so.

The required impedances in Eq. (2) are well known but the radiation factor linking the radiated power by the action of the sound bridge and its velocity will represent a problem if the frequency range of interest extends well above the critical frequency f_c of one or both plates. (It should be noted that the definition of σ_B differ from the common radiation factor of plates based on the time- and space-averaged velocity of the plate).

Approximate expressions for the radiation factor are well known for $f << f_c$ but to the author's knowledge no simple and correct expressions exists for line-driven plates, neither near to the critical frequency nor above. Using a variable M equal to $(f/f_c)^{1/2}$, Innes and Crighton[5] derived approximate expressions for the radiated power from an infinite line-driven fluid-loaded plate covering three ranges, $0 < M < 1$, $M \approx 1$ and $M > 1$. However, by neglecting the inner energy losses in the plate, the expressions for the latter two ranges are not physically meaningful. In the next section a new integral expression is presented based on their results, however, no approximate expressions are given.

B. Sound power radiated from line- and point-driven plates

For an infinite plate of mass per unit area m and bending stiffness B driven along a line by a force of amplitude F_0 per unit length, Innes and Crighton[5] derived the following expression for the normalized (or relative) radiated sound power

$$W_{\mathrm{rel}} = \frac{W}{W_{\mathrm{mec}}} = \frac{4}{\pi} \cdot \frac{\rho_0 \omega^4 m}{B^2 k_p} \int_0^{\pi/2} \frac{\sin^2\theta}{\left[k_0^2 \sin^2\theta (k_0^4 \cos^4\theta - k_p^4)^2 + \mu^2 k_p^8\right]} \mathrm{d}\theta, \tag{3}$$

normalized to the input mechanical power (without fluid loading) given by

$$W_{\mathrm{mec}} = \frac{|F_0|^2 k_p}{8\omega m}. \tag{4}$$

In these equations, k_0 and k_p are the wavenumber in the surrounding medium and the plate free surface wavenumber, respectively, the latter given by $k_p = (m\omega^2/B)^{1/4}$. The variable μ is the ratio of the fluid density ρ_0 and the surface mass m, i.e. a variable characterizing the fluid loading.

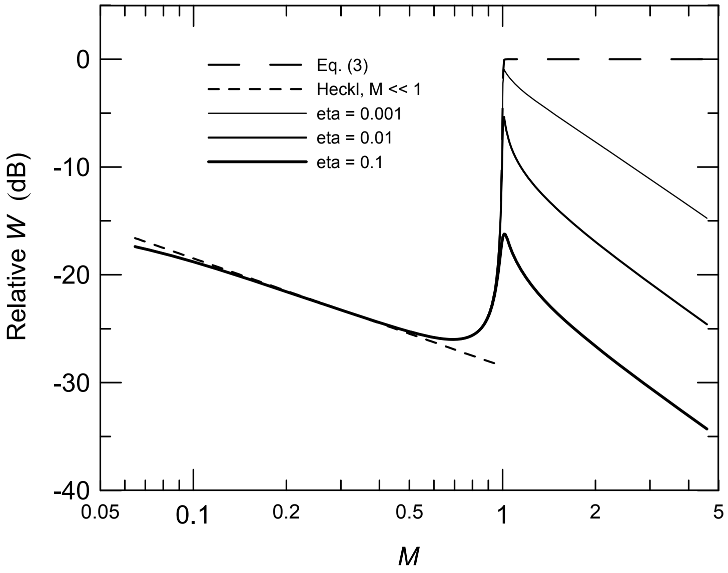

Figure 1: Relative sound power radiated from an infinite 5 mm thick steel plate in air driven along a line. Numerical result using Eq. (3) (equivalent to Eq. (2.8) by Innes and Crighton[5]), together with results with different loss factors η by modifying Eq. (3) with Eq. (5). "Classical" prediction by Heckl[6,7] is also shown.

Evaluating this integral numerically, for a 5 mm thick plate of steel with air loading is shown in Figure 1 using, in the same way as Innes and Crighton[5], the variable $M = k_0/k_p = (f/f_c)^{1/2}$ as the abscissa. The "classical" result for $M << 1$, first given by Heckl[6,7], is also shown. The problem, however, is that for $M \geq 1$, the total input mechanical power is radiated as sound, a result which cannot physically be correct.

We shall therefore have to develop a new integral equation by introducing a loss factor η in the bending stiffness, i.e. setting $B = B(1 - \mathrm{i}\eta)$ or equivalently, $k_p^4 = k_p^4/(1 - \mathrm{i}\eta)$. The relative power then comes out with an added "loss" term L_η in the denominator in Eq.(3), given by

$$L_\eta = \eta k_0^5 \sin\theta \cos^4\theta \left[2\mu k_p^4 + \eta k_0^5 \sin\theta \cos^4\theta\right]. \tag{5}$$

The result when evaluating the integral in Eq.(3) with this added term, is shown in Figure 1 for three values of η, being 0.1, 0.01 and 0.001. As for applications in building acoustics, the range between 0.01 and 0.1 will be a realistic one.

Analog results, applicable for a point-driven, infinitely large plate, will also be presented. These are calculated applying the expression for the far-field sound pressure given by Junger and Feit[8], see their Eq. (8.30).

3. MEASURED AND PREDICTED RESULTS

Applying the simple correction outlined in section 2.A above presupposes that the structural bridges are mass-less and infinitely stiff. Even when assuming the bridges to be infinitely stiff, a practical problem concerning line-connections (studs or purlins) is whether they may be treated as point- or line-connectors. Normally, the plates are not glued to the studs but fastened by screws which may act like discrete contact points.

This effect is clearly demonstrated by Craik and Smith[9] and also in an extensive series of measurements by Hongisto et al.[10] on an experimental double-leaf construction where the leaves were 2 mm steel plates. A number of different steel studs as well as wooden studs were tested, using variables such as the cavity depth, the cavity infill and the screw spacing. All specimens were, however, mounted in an opening of dimensions 1105 x 2250 mm, i.e. with an area of about one-forth of the 10 m^2 area normally used in laboratory tests on sound insulation of walls.

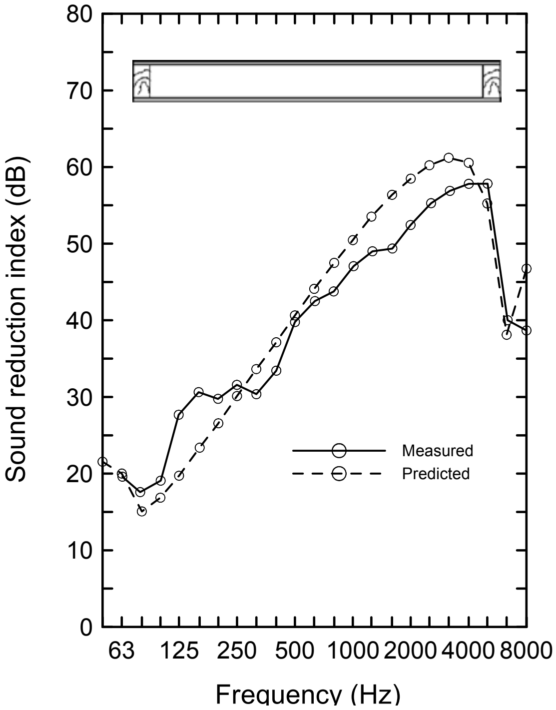

Figure 2: Sound reduction index of a double-leaf construction of 2 mm steel plates with an empty cavity of thickness 84 mm. Wooden studs with a centre-to-centre distance of 1100 mm. Measured results reproduced from Fig. 12b in reference[10].

One example from this series of measurements is shown in Figure 2, having the smallest screw spacing (170 mm) to compare with predictions assuming line-drive. The example is the same as used by Wang et al.[2] in their Figure 14 to compare their models with experimental data. The measurement data in Figure 2 is plotted together with the data using the present prediction model assuming line-drive conditions. The studs were wooden with a spacing of 1100 mm and the cavity of depth 84 mm was empty, see inserted sketch. Material parameters are identical to the ones given in reference[10], however, to introduce some energy losses in the air-filled cavity, the air layer in the transfer matrix model is assumed to have a power attenuation coefficient equal to 0.2 m^{-1}. The predicted curve using the periodic model from reference[2] is not plotted for comparison in this figure as it has a “wavy” shape due to strong wave reflections in the highly stiff studs. Unfortunately, no attempt was made by the authors to present the data averaged in one-third-octave bands as used in the examples here.

The second example illustrates the effect of a so-called acoustical lining, a panel with high coincidence frequency attached to a massive wall. The massive wall is made of lightweight concrete blocks of density 790 kg/m^3 with a 10 mm plaster layer and the lining is 13 mm plasterboard attached to the wall by studs of 37mm × 50mm, i.e. with a cavity of depth 50 mm, here filled with an absorbent of density 33 kg/m^3 and estimated flow resistivity of 12 kPa·s/m^2. The distance between the studs is 600 mm. (Data and measurement results are collected from an unpublished report by the Acoustic laboratory, NTH, Trondheim).

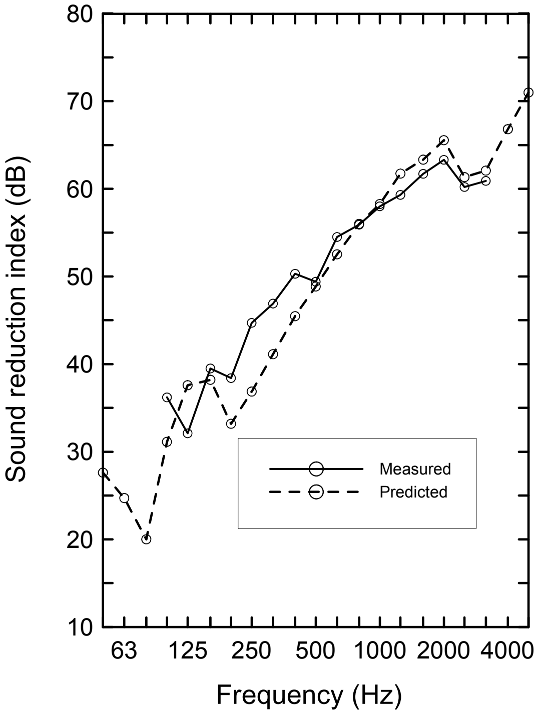

Figure 3: Measured and predicted sound reduction index of a massive wall with an acoustic lining of 13 mm plasterboard. Massive wall is constructed from 150 mm thick blocks of lightweight concrete with a layer of plaster. The lining is fixed to the wall by 35×50 mm wooden studs. The cavity of 50 mm is filled with an absorbent of density 33 kg/m^3.

Although the thickness of the massive wall is 150 mm, we have for simplicity applied a thin plate model in the predictions; the resulting curves, predicted as well as measured, for the wall with the lining are shown in Figure 3. Apart from the discrepancies in the frequency range 200–500 Hz, the fit between measured and predicted results is reasonably good. It is also interesting to note that the effect of the lining is quite well predicted; i.e. by comparing the sound reduction index of the wall with lining and the bare massive wall. This is shown in Figure 4, where the measured and predicted improvement offered by the lining is plotted, also showing (by the dashed line) the maximum achievable improvement as calculated using the simple formula for rigid line connections due to Heckl[6,7]. The latter may be written

$$\Delta R = -10\lg\left[\frac{2c_0}{\pi l\, f_c}\right], \qquad (6)$$

where *l* is the distance between the studs. From the material data given for the plasterboard lining the critical frequency is approximately 2800 Hz.

It should be noted that in the examples shown here, a spatial windowing technique based on the work by Villot and Guigou-Carter[11] is used, however applying a simplified one-dimensional approach, see Vigran[12].

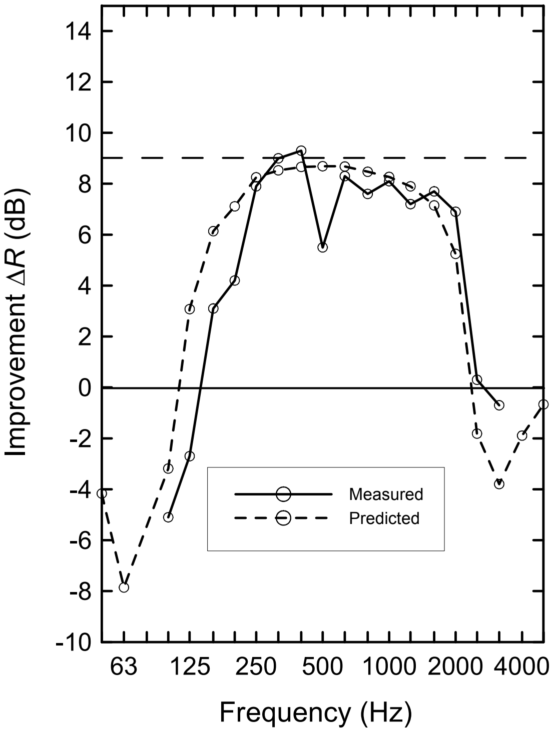

Figure 4: Measured and predicted sound reduction improvement by the acoustical lining described in Fig. 3. Dashed line – maximum improvement predicted by Eq. (6).

4. CONCLUSIONS

To account for the effect of structural connections (sound bridges) in double walls the semi-empirical method proposed by Sharp[3] is revived and used to correct resulting data from the transfer matrix method calculation on the construction without bridges. Usually, only lightweight double-leaf constructions have been the subjects when modeling the effect of such bridges. By including heavier constructions implies that larger part of the actual frequency range will be above the critical frequency of the constituent parts. Evaluating the radiation factor for an extended frequency range, both for line- and point-drive, was therefore necessary. Different constructions are used to compare measured and predicted results, of which two are presented here. In spite of the assumptions, such as mass-less and infinitely stiff bridges together with bridge radiation factors for infinite size plates, the predictions are reasonably good.

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